Math 151 - Exam 1B - solutions

Problem 1 [12 pts] Find all solutions: $2^{x^2+3x} = 16$.
Solution Take logarithms with base 2 to get $x^2 + 3x = 4$, so $x = -4$ or $x = 1$ by the quadratic formula.

Problem 2 [15 pts] Solve for $t$. First, write in logarithmic form, then round the answer to four decimal places.

$$4^t = 5.$$ 

Solution The logarithmic form of this equation is

$$t \log 4 = \log 5.$$ 

The answer is then

$$t = \frac{\log 5}{\log 4} \approx 1.1610.$$ 

Problem 3 [5 pts] Let $\log_b A = 4$ and $\log_b B = -3$. Find the value of the following:

$$\log_b \left( \frac{\sqrt{A}}{B^2} \right)$$ 

Solution Call this number $x$. Using properties of logarithms,

$$x = \log_b(\sqrt{A}) - \log_b(B^2) = \frac{\log(A)}{2} - 2 \log_b(B) = 2 + 6 = 8.$$ 

Problem 4 [10 pts] Find the interest earned on $80,000 invested for 20 years at 6.5% interest, compounded semi-annually (twice a year). Round to the nearest cent.
Solution The whole amount you have in the bank is

$$80,000 \cdot 1.0325^{40} \approx 287536.11$$ 

For the interest earned, subtract the capital. So the answer is

$$287536.11 - 80,000 = 207536.11.$$
**Problem 5** [15 pts] Consider the function \( f(x) = x^2 + 4x - 3 \).

a) Find the average rate of change \( f_{2,4} \) for \( f \) between 2 and 4.

b) Write \( f_{2,b} \) for the average rate of change for \( f \) between 2 and \( b \). Find the limit \( L = \lim_{b \to 2} f_{2,b} \).

**Solution**

a) We use \( f(2) = 9 \) and \( f(4) = 29 \). So

\[
f_{2,4} = \frac{29 - 9}{4 - 2} = 10.
\]

b) First,

\[
f_{2,b} = \frac{f(b) - f(2)}{b - 2} = \frac{b^2 + 4b - 12}{b - 2}.
\]

Factor the numerator, cancel out \( b - 2 \), and you get that the instantaneous rate of change is

\[
\lim_{b \to 2} \frac{b^2 + 4b - 12}{b - 2} = \lim_{b \to 2} b + 6 = 8.
\]

**Problem 6** [10 pts] Find (briefly explain your answer) \( \lim_{z \to \infty} \frac{4z^2 + 2z - 1}{3z^2 - 4z + 1} \).

**Solution** The degree of the numerator (= 2) is the same as that of the denominator, and the coefficients of the highest powers are 4 and 3, respectively. So the limit is \( \frac{4}{3} \).

**Problem 7** [8 pts] Let \( f(x) = \begin{cases} 4 - x^2 & \text{if } x < 1 \\ x + 2 & \text{if } 1 \leq x < 5 \\ x - 2 & \text{if } 5 \leq x \end{cases} \)

Find the following limits. Write DNE for a limit that does not exist.

a) \( \lim_{x \to 1^-} f(x) = 3 \)

b) \( \lim_{x \to 1^+} f(x) = 3 \)

c) \( \lim_{x \to 1} f(x) = 3 \)

d) Find all points \( x \) where \( f(x) \) is not continuous.

**Solution** Plotting this function may help to see what is going on, see above.

d) \( f(x) \) is continuous at all points \( x \) except at \( x = 5 \). Note that for \( x = 1 \), both one-sided limits agree with \( f(1) = 3 \), so \( f(x) \) is continuous there. All other \( x \)-values besides 1 and 5 are using only one (polynomial) formula for \( f(x) \), so \( f(x) \) is continuous there.
**Problem 8** [15 pts] Suppose $G(t)$ is the number of US households with an iGadget. Assume $G(t)$ is modeled with an exponential law,

$$G(t) = Ce^{kt}.$$ 

Suppose $G(0) = 5,000$ in the year 2010 (at time $t = 0$). Four years later, $G(t)$ has increased to 9,000.

a) Find the values of the constants $C$ and $k$. Give exact answers (no decimal fractions).

b) In what year will the number of US households with an iGadget be 1 million, assuming $G(t)$ keeps following the same law? Round your answer to one digit after the decimal point.

**Solution**

a) We get $C = 5,000$ from $t = 0$. Now substitute $t = 4$ and $G(4) = 9,000$. Solve this for $k$ by taking logarithms to get

$$k = \frac{\ln(9/5)}{4} = \frac{\ln 1.8}{4}.$$ 

b) We need to use the values for $C$ and $k$ from part a) and solve $G(t) = 1,000,000$ for $t$. This gives

$$200 = e^{kt}$$

We take logarithms to get $\ln 200 = kt$. Divide by $k$ to get

$$t = \frac{\ln 200}{k} = \frac{4 \ln 200}{\ln 1.8} \approx 36.1$$

(so the year is approximately 2046).

**Problem 9** [10 pts] Below is a graph of real gross domestic product $GDP(t)$ of the United States in billions of 2006 dollars (source: Federal Reserve St. Louis).
Estimate the average rate of change per year of $GDP(t)$ between 2010 and 2014. Round the answer to one digit after the decimal point.

**Solution** We have to read the values of $GDP$ off the graph. This is of course not very accurate, we’ll just say

$$GDP(2010) \approx 14,600$$
$$GDP(2014) \approx 15,800$$

$$GDP_{2006,2010} = \frac{15800 - 14600}{2014 - 2010} = 300.$$

This rate of change is in billions of dollars per year.