Lec. 13: 2-11-09

1. Review

2. Application to viscous slot flow (VSF)

Review

We solved VSF with a force balance: pressure ~ viscous
then we derived the general con. eqns (mass & mom)

Cons. of mass
1. Check a flow
2. Compute one velocity component & then others
3. Relate velocity scales & length scales
Cons. of mom
1. Check a flow
2. Compute pressure given velocity
3. Solve for a flow.

Solution method
1. Physics: State the problem
2. Math: Develop a mathematical statement \( \text{(Cons. laws)} \), \( \text{(BC, simplifications)} \)
3. Physics: Approximate
4. Math: Solve
5. Physics: Check & interpret
Viscous slot flow (§9.4)

1. State problem

Compute flow rate $Q'$ per unit width in long, thin slot with constant pressure gradient.

To compute flow rate, need velocity.
To compute velocity, solve cons. of mom.

\[
\begin{align*}
H &= 0.01 \text{ m} \\
L &= 1 \text{ m} \\
-\frac{\partial p}{\partial x} &= 150 \text{ Pa/m} \\
\text{Glycerin} &:
\begin{align*}
\mu &= 1.5 \text{ Pa·s} \\
\rho &= 1300 \text{ kg/m}^3 \\
v &= 1.2 \times 10^{-3} \text{ m}^2/\text{s}
\end{align*}
\end{align*}
\]
2. Write math statement

x-mom.

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \]

unsteady, advective, pressure, viscous, inertial

Boundary conditions: \( u = 0 \) at \( y = 0, H \) (no slip)
3. Approximate

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \]

Compare terms

\[ \frac{\nu \frac{\partial^2 u}{\partial y^2}}{\nu \frac{\partial^2 u}{\partial x^2}} \sim \sqrt{\frac{h}{L^2}} = (\frac{L}{H})^2 = (\frac{1m}{0.01m})^2 \]

\[ = 10^4 \]

Neglect \( \nu \frac{\partial^2 u}{\partial x^2} \) with respect to \( \nu \frac{\partial^2 u}{\partial y^2} \)
Examine inertia

\[ \frac{u \partial u}{\partial x} \sim \frac{u^2}{\nu} = \frac{U H}{L} \frac{H}{L} \to \text{Expect this to be small} \]

\[ \uparrow \]

Reynolds number

Compare pressure of \( y \)-viscous, keep both

\[ -\frac{1}{\rho} \frac{\partial p}{\partial x} \sim \nu \frac{\partial^2 u}{\partial y^2} \]

\[ \nu \frac{U}{H^2} \sim -\frac{1}{\rho} \frac{\partial p}{\partial x} \]

\[ U \sim -\frac{H^2 \frac{\partial p}{\partial x}}{\rho \nu} = \frac{-h^2 \frac{dp}{dx}}{\mu \frac{dx}{dx}} = \frac{(0.01 \text{m})^2}{1.5 \text{Re} \cdot s} (150 \text{Pa/m}) \]

\[ = 0.01 \text{ m/s} \]

\[ \frac{U H}{\sqrt{L}} = \frac{(0.01 \text{m/s})(0.01 \text{m})^2}{(1.2 \times 10^{-3} \text{ m}^2/\text{s})(1 \text{m})} \sim 10^{-3} \]