Lec. 11: 2-6-09

1. Salt fingers, internal waves
2. Stress & cons. of momentum

Salt fingers

2°F

Warm, fresh
cold, salty

Gravitationally stable

Unstable

Depends: If the temp. gradient is large enough, then it could be stable.
Another instability

Heat can diffuse into pipe
But salt can’t

Warm water rises & flows out of the pipe

No pipe necessary!

Diffusion of heat $\Rightarrow$ diffusion of salt

$T$ $S$

Gets warmer before it gets saltier

As warm but less salty than the water at the same level $\Rightarrow$ rises

Fingering convection
Stress in a moving fluid (§§ 3.6, 3.7; §§ 4.10, 11)

Solid: Stress $\propto$ strain

Fluids: Stress $\propto$ rate of strain

(Newtonian fluid - water, air)

Stretch a wire by applying force
Measure elongation
Strain $\propto \frac{1}{E}$

slope $= \frac{1}{E}$

$E =$ modulus of elasticity
Consider 1D shear flow

\[ y = y_0 + \Delta y \]
\[ u(y) \]
\[ \frac{\partial u}{\partial y} \Delta y \]
\[ \Delta x \rightarrow \Delta x \rightarrow 1 \]

Rate of angular strain = \( \frac{\partial \Theta}{\partial t} = \lim_{\Delta t \to 0} \frac{\partial \Theta}{\partial t} \)

\[ \tan \Theta \approx \Theta = \frac{(u(y_0 + \Delta y) - u(y_0)) \Delta t}{\Delta y} \]

\[ \frac{\partial \Theta}{\partial t} = \frac{u(y_0 + \Delta y) - u(y_0)}{\Delta y} \]

\[ \frac{d \Theta}{dt} = \frac{\varepsilon u}{\varepsilon y} \]
Generalize

\[ \mathbf{v} \rightarrow \begin{array}{c}
\mathbf{u}
\end{array} \rightarrow \begin{array}{c}
\mathbf{u}'
\end{array} \quad \begin{array}{c}
\mathbf{e}_1
\end{array} \rightarrow \begin{array}{c}
\mathbf{e}_2
\end{array} \]

Rate of ang strain \( = \frac{d\Theta_1}{dt} + \frac{d\Theta_2}{dt} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \)

For water & air, stress \( \propto \) rate of strain

\( \tau_{y} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \)

\( \tau_{x} = \mu \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \)

\( \tau_{y} = \mu \left( \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} \right) - \rho \)
\[ \iiint \text{\textit{\varepsilon}} \frac{Dw}{Dt} = -sg + \frac{\partial w}{\partial x} + \frac{\partial \nu w}{\partial y} + \frac{\partial w^2}{\partial z} \]

\[ = -sg + \frac{\partial}{\partial x} \left[ \mu \left( \frac{\partial w}{\partial x} + \frac{\partial w}{\partial z} \right) \right] + \frac{\partial}{\partial y} \left[ \mu \left( \frac{\partial w}{\partial z} + \frac{\partial w}{\partial y} \right) \right] + \frac{\partial}{\partial z} \left[ \mu \left( \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} \right) \right] - \rho \]

\[ \iiint \frac{Dw}{Dt} = -sg + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - \frac{\partial \rho}{\partial z} \]

\[ + \frac{\partial}{\partial x} \left[ \frac{\partial w}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right] \]

\[ = 0 \text{ for incompressible flow} \]