3. If $f(x)$ is a positive definite quadratic, show that the exact Newton step for minimizing $f$ satisfies the sufficient decrease assumption (3.4) for any $c_1 \leq \frac{1}{2}$ and the curvature condition (3.5) for any $c_2 > 0$.

4. Let $f(x) = x_1^4 + x_2^2$ and $x_0 = [1 1]^T$. Find $x_1 = x_0 + \alpha_0 p_0$ in the following cases.
   a) $p_0$ is the steepest descent direction and $\alpha_0$ is the result of an exact line search.
   b) $p_0, \alpha_0$ are found by taking an exact Newton step
   c) $p_0$ is the Newton step direction and $\alpha_0$ is the result of an exact line search in this direction.
   (Doing an 'exact' line search here may require you to solve a higher degree polynomial equation – in such a case you should approximate $\alpha_0$ to high accuracy using some one dimensional root finder, such as Newton’s method.)

5. For the same problem as in #4, find $p_0$ using the trust region method with trust region radius $\Delta_0 = 1$ and the exact Hessian for $B_0$. Also find the corresponding 'dogleg step' approximation to $p_0$.

6. If $B$ is a symmetric positive definite matrix, $g$ is a vector, $s(\lambda) = (B + \lambda I)^{-1}g$, and $\eta(\lambda) = ||s(\lambda)||^2$, show that

$$
\eta'(\lambda) = -\frac{1}{2} s(\lambda)^T (B + \lambda I)^{-1} s(\lambda) \quad \lambda > 0
$$

(This is what we would need to solve $||(B + \lambda I)^{-1} g|| = \Delta$ for $\lambda$ by Newton’s method.)