1. (Data fitting) If data points \((x_1, y_1), \ldots, (x_n, y_n)\) are given, consider the problem of finding a polynomial of degree \(m\)

\[
P_m(x) = \sum_{k=0}^{m} a_k x^k
\]

passing through these \(n\) points. If \(m < n - 1\) then in general no solution will exist, since this amounts to \(n\) conditions for \(m + 1\) unknowns. Instead we may interpret the problem as that of solving the linear system

\[
\sum_{k=0}^{m} a_k x_j^k = y_j \quad j = 1, \ldots, n
\]

in the least square sense. Find the normal equations which may be used to solve for \(a_0, \ldots, a_m\) for \(m = 1, 2\).

2. (Newton’s method in the complex case) Newton’s method

\[
z_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)}
\]

for root finding can be justified for functions \(w = f(z)\) of a complex variable, by more or less the same argument as in the real case. Use it, with some sensible initial guesses, to find all complex roots of \(f(z) = z^3 + 2z^2 + 3z + 4\) to at least six significant figures.