6. Find the general solution of $(1 + x^2)u_x + u_y = 0$. Sketch some of the characteristic curves.

7. Find the solution of

$$yu_x + xu_y = 1 \quad u(0, y) = e^{-y^2}$$

Discuss why the solution you find is only valid for $|y| \geq |x|$.

8. Find the general solution of $u_{xx} - 4u_{xy} + 3u_{yy} = 0$.

9. Find the regions of the $xy$ plane where the PDE

$$yu_{xx} - 2u_{xy} + xu_{yy} - 3u_x + u = 0$$

is elliptic, parabolic, and hyperbolic.

10. Carry out the details of showing that in polar coordinates $(r, \theta)$ Laplace’s equation in $\mathbb{R}^2$ becomes

$$u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta \theta} = 0$$