MATH 501 FALL 2006 Supplementary Homework 5

1. Let $\{f_n\}$ be a sequence of continuous functions on $[a, b]$ converging uniformly to f on $[a, b]$. Show that there exists a constant M such that $|f_n(x)| \leq M$ for all $x \in [a, b]$ and all n. Show by example that the conclusion may be false if f_n converges pointwise but not uniformly.

2. Let

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{x^2 + n^2}$$

Prove that $f \in C^1(\mathbb{R})$ and find (with justification) a series formula for $f'(x)$.

3. If f is uniformly continuous on $\mathbb{R}$ and $f_n(x) = f(x + \frac{1}{n})$ show that $f_n \to f$ uniformly on $\mathbb{R}$.

4. The *error function* is defined by

$$\text{erf}\,(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-s^2}\, ds$$

Find (with justification) a power series formula for $\text{erf}\,(x)$. Where does it converge?