1. Let $f \in C^3(a, b)$ and $x \in (a, b)$.

a) Show that there exists $\delta_1, C_1$ such that

$$\left| \frac{f(x + h) - f(x)}{h} - f'(x) \right| \leq C_1 h \quad |h| < \delta_1$$

b) Show that there exists $\delta_2, C_2$ such that

$$\left| \frac{f(x + h) - f(x - h)}{2h} - f'(x) \right| \leq C_2 h^2 \quad |h| < \delta_2$$

c) Discuss why is the symmetric difference $\frac{f(x + h) - f(x - h)}{2h}$ will typically be a better approximation to $f'(x)$ than the usual divided difference $\frac{f(x + h) - f(x)}{h}$.

(Suggestion: In parts a) and b) use Taylor’s theorem.)

2. Let $f(x) = \sqrt{x}$ and $\alpha = 1$.

a) Compute the Taylor polynomials $P_1, P_2, P_3, P_4$ for $f(x)$ at $\alpha$.

b) Estimate the error in approximating $\sqrt{1.2}$ using $P_4(x)$, and compare to the actual error.