MATH 501 FALL 2006 Supplementary Homework 3

1. Let $f \in C^3(a,b)$ and $x \in (a,b)$.

a) Show that there exists δ_1, C_1 such that

$$\left| \frac{f(x+h)-f(x)}{h} - f'(x) \right| \leq C_1 h \qquad |h| < \delta_1$$

b) Show that there exists δ_2, C_2 such that

$$\left| \frac{f(x+h)-f(x-h)}{2h} - f'(x) \right| \leq C_2 h^2 \qquad |h| < \delta_2$$

c) Discuss why is the symmetric difference $\frac{f(x+h)-f(x-h)}{2h}$ will typically be a better approximation to $f'(x)$ than the usual divided difference $\frac{f(x+h)-f(x)}{h}$.

(Suggestion: In parts a) and b) use Taylor's theorem.)

2. Let $f(x) = \sqrt{x}$ and $\alpha = 1$.

a) Compute the Taylor polynomials P_1, P_2, P_3, P_4 for $f(x)$ at α.

b) Estimate the error in approximating $\sqrt{1.2}$ using $P_4(x)$, and compare to the actual error.