MATH 501 FALL 2006 Supplementary Homework 2

1. If $f(x) = \dfrac{1}{x^2}$ find $f'(x)$ directly from the definition of derivative.

2. Let $f(x) = \begin{cases} x^2 & x \in \mathbb{Q} \\ 0 & x \in \mathbb{Q}^c \end{cases}$. Show that $f$ is differentiable at $x = 0$ and find $f'(0)$.

3. Let

$$f_s(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

(sometimes called the *symmetric derivative* of $f$ at $x$).

a) If $f$ is differentiable at $x$ show that $f'(x) = f_s'(x)$.

b) Show by example that $f_s'(x)$ may exist even if $f$ is not differentiable at $x$.