

1. (20 points) Find the solution of

$$y' = \frac{y^4}{e^{2x}} \quad y(0) = 1$$

What is the existence interval for this solution?

2. (15 points) Find the general solution of $x' + \frac{6}{t}x = t^2 - 1$.

3. (15 points) Use the substitution $v = \ln y$ to find the general solution of the equation

$$y' + e^t y = y \ln y$$

4. (20 points) If $y_1(x) = e^x$, $y_2(x) = x$, verify that $\{y_1, y_2\}$ is a fundamental set for the ODE

$$(x - 1)y'' - xy' + y = 0$$

Find the solution of this equation satisfying $y(2) = 1$, $y'(2) = 0$.

5. (15 points) Find the general solution of

$$2y'' + y' + 4y = 0$$

6. (15 points) The acceleration of a certain sports car is proportional to the difference between 250 km/h and its velocity.

a) What is the differential equation satisfied by the velocity?

b) If it takes 10 seconds to accelerate from rest to 100 km/h, how long does it take to accelerate from rest to 200 km/h?