Exam 3 answers

1. $\mathbf{y}(t) = C_1 e^t \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + C_2 e^t \begin{bmatrix} 1/4 \\ t - 7/4 \\ -t \end{bmatrix} + C_3 e^{2t} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Comments: (i) Remember that for a triangular matrix the eigenvalues are the same as the diagonal entries. (ii) When you compute the second term in the form $e^t(\mathbf{v}_2 + \mathbf{v}_1 t)$, you *cannot* multiply $\mathbf{v}_2$ by any scalar – that is only true when solving a homogeneous system, as in the case of an eigenvector.)

2. If $y_1 = x, y_2 = x'$ then the equivalent first order system is

$$\mathbf{y}' = \begin{bmatrix} 0 & 1 \\ -9 & -2 \end{bmatrix} \mathbf{y}$$

The eigenvalues of the coefficient matrix are $\lambda = -1 \pm \sqrt{8}i$ so that 0 is an asymptotically stable spiral point.
Comment: You should be able to guess the answer to part b) without answering a): This is underdamped motion, so that $x(t)$ must cross 0 infinitely often, and the only type of phase portrait with that property is the spiral point. Also damping means that the amplitude of the oscillation decays to 0, so 0 must be asymptotically stable.)

3. a) A fundamental set is

$$e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad e^{-t} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

and the corresponding fundamental matrix is

$$Y(t) = \begin{bmatrix} e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$$

b) $\mathbf{y}(t) = \begin{bmatrix} 5e^t - e^{-t} \\ 5e^t - 3e^{-t} \end{bmatrix}$ c) $\mathbf{y}_p(t) = \begin{bmatrix} (1/2 - t)e^t \\ (3/2 - t)e^t \end{bmatrix}$

4. a) $(0, 0)$ and $(4, 2)$.

b) $(0, 0)$ is an asymptotically stable spiral point and $(4, 2)$ is a saddle point (so automatically unstable).