1. The electron in a hydrogen atom has the following wave function

\[ \psi = A R_{21} \left( \left| \frac{3}{2} \right|^1 \right) - \left| \frac{1}{2} \right|^1 \right) \]

where the ket vectors are given in the \(| j m_j \rangle\) basis.

(a) Determine the normalization constant \(A\). (2 points)

(b) If you measure the total angular momentum squared, \(J^2\), what values might you get and what is the probability of each? (3 points)

(c) If you measure the \(z\) component of the orbital angular momentum, \(m_l\), what values might you get and what is the probability of each? (3 points)

(d) If you measure the \(x\) component of the electron spin, \(S_x\), what values might you get and what is the probability of each? (2 points)

2. The (unperturbed) Hamiltonian \(H^0\) of a three state system is given in the basis

\[
\begin{align*}
\psi_1 &= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\
\psi_2 &= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \\
\psi_3 &= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}
\end{align*}
\]

as

\[ H^0 = V_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

A perturbation of the form

\[ H' = \epsilon \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix} \]

is introduced with \(\epsilon \ll V_0\).

(a) What are the energy levels in zeroth-order perturbation theory? (1 points)

(b) What are the energy levels in first-order perturbation theory? (3 points)

(c) What are the corresponding good states in zeroth-order perturbation theory? (3 points)

(d) If the system is in the state \(\psi_1\) at time \(t=0\), what is the (time-dependent) probability to find it in state \(\psi_3\) at a later time \(t'\)? (3 points)

3. Two spin-\(\frac{1}{2}\) particles interact via the Hamiltonian \(\vec{S}_1 \cdot \vec{S}_2\). Assume that only the spin degrees of freedom are relevant for this problem.

Use the direct product basis \(|s_1 s_z,1\rangle|s_2 s_z,2\rangle\) when an explicit basis is required. You can use the ordered arrow notation in which e.g. \(\uparrow\downarrow \equiv |\frac{1}{2} \frac{1}{2} \rangle |\frac{1}{2} -\frac{1}{2} \rangle\).

(a) Write down the operator \(Q = S_{1y} S_{2y}\) in matrix representation in this basis. (3 points)

(b) Write the wave function for this system such that if \(S_{1z}\) is measured, the probability of obtaining \(\hbar/2\) is 1, but if \(S_{2z}\) is measured, the probability of obtaining \(\hbar/2\) is 50%. (2 points)

(c) Calculate the expectation value \(\langle Q \rangle\) for the state in part (b). (3 points)

(d) Show that the eigenstates of \(Q\) do not have definite energy. (2 points)

Good luck!