This approach is due to Box and Muller. Let us prove that, if $U_1$ and $U_2$ are independent random variates from $\mathcal{U}(0, 1)$, then the variates

$$Z_1 = (-2 \ln U_1)^{1/2} \cos 2\pi U_2$$

$$Z_2 = (-2 \ln U_1)^{1/2} \sin 2\pi U_2$$

are independent standard normal deviates. To see this let us rewrite the system \((1)\) as

$$Z_1 = (2V)^{1/2} \cos 2\pi U$$

$$Z_2 = (2V)^{1/2} \sin 2\pi U,$$

where $V$ is from $\exp(1)$ and $U_2 = U$. It follows from \((2)\) that

$$Z_1^2 + Z_2^2 = 2V \quad \text{and} \quad \frac{Z_2}{Z_1} = \tan 2\pi U.$$

The Jacobian of the transformation

$$J = \begin{vmatrix}
\frac{\partial u}{\partial z_1} & \frac{\partial u}{\partial z_2} \\
\frac{\partial v}{\partial z_1} & \frac{\partial v}{\partial z_2}
\end{vmatrix}
= \begin{vmatrix}
-z_2 \cos^2 2\pi u & \cos^2 2\pi u \\
2\pi z_1 & 2\pi z_1
\end{vmatrix}
= \begin{vmatrix}
-z_2 & z_1 \\
4\pi v & z_1
\end{vmatrix}
= -\frac{1}{4\pi v} (z_2^2 + z_1^2) = -\frac{1}{2\pi}
$$

and

$$f_{Z_1, Z_2}(z_1, z_2) = f_{U, V}(u, v)|J| = \frac{1}{2\pi} \exp\left(-\frac{z_1^2 + z_2^2}{2}\right).$$

The last formula represents the joint p.d.f. of two independent standard normal deviates.