

where  $\pi_j$  is probability of a rat in Group  $j$  having One or More Tumors.  
 $\chi^2 = 3.312$  with  $df = (2-1)(3-1) = 2$  and  $p$ -value = 0.191.

c. No

10.81 a. The results are summarized in the following table:

Question	$\hat{\pi}$	$\hat{\sigma}_{\hat{\pi}}$	95% C.I.
Did Not Explain?	0.254	0.01947	(0.216, 0.292)
Might Bother?	0.916	0.0124	(0.892, 0.940)
Did Not Ask?	0.471	0.02232	(0.427, 0.515)
Drug Not Changed?	0.877	0.0147	(0.848, 0.906)

10.85  $\bar{y} = \sum_i (No.Mites)(frequency)/500 = 1.146$

we obtain the following using a Poisson distribution with  $\mu = 1.2$ :

Mites/Leaf ( $k_i$ )	0	1	2	3	4	$\geq 5$	Total
$\pi_i = P(y = k_i)$	0.3012	0.3614	0.2169	0.0867	0.0260	0.0078	1.0
$E_i = 500\pi_i$	150.6	180.7	108.5	43.3	13	3.9	500
$n_i$	233	127	57	33	30	20	500

$\chi^2 = 176.6$  with  $df = 6-1 = 5$ ,  $\Rightarrow p$ -value =  $Pr(\chi^2 > 176.6) < 0.001 \Rightarrow$   
 Reject  $H_o$

## Chapter 11: Linear Regression and Correlation

11.3 The calculations are give here:

$i$	$x_i$	$y_i$	$(x_i - 3)^2$	$(x_i - 3)(y_i - 5.6)$
1	1	2	4	7.2
2	2	4	1	1.6
3	3	6	0	0
4	4	7	1	1.4
5	5	9	4	6.8
Total	15	28	10	17

$$\bar{x} = 15/5 = 3 \quad \bar{y} = 28/5 = 5.6$$

$$S_{xx} = \sum_i (x_i - 3)^2 = 10$$

$$S_{xy} = \sum_i (x_i - 3)(y_i - 5.6) = 17$$

$$\hat{\beta}_1 = S_{xy}/S_{xx} = 17/10 = 1.7$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = 5.6 - (1.7)(3) = 0.5$$

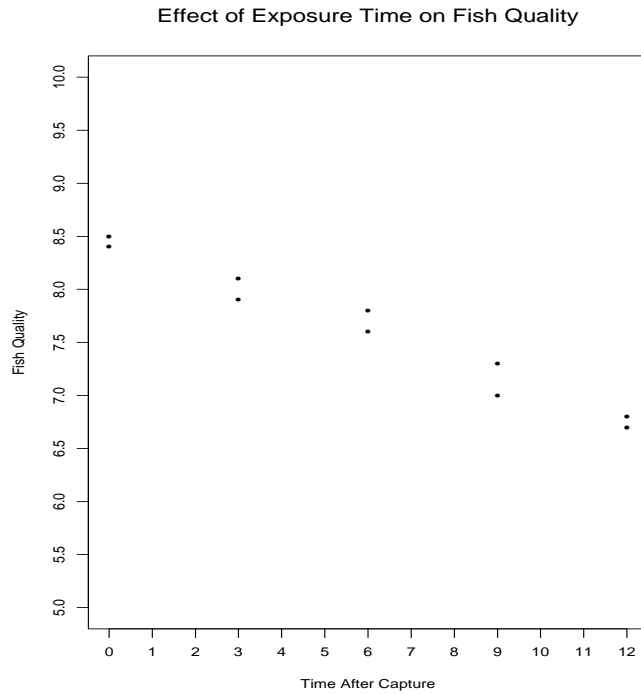
$$\hat{y} = 0.5 + 1.7x$$

11.5  $\hat{\beta}_1 = 1.633$

$$\hat{\beta}_0 = 2.468$$

$$\hat{y} = 2.468 + 1.633x$$

11.9 a. A scatterplot of the data is given here:



b. Minitab output is given here:

Regression Analysis: QUALITY versus STORAGE

The regression equation is  
 QUALITY = 8.46 - 0.142 STORAGE

Predictor	Coef	SE Coef	T	P
Constant	8.46000	0.06610	127.99	0.000
STORAGE	-0.141667	0.008995	-15.75	0.000

S = 0.1207      R-Sq = 96.9%      R-Sq(adj) = 96.5%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	3.6125	3.6125	248.07	0.000
Residual Error	8	0.1165	0.0146		

Total                    9            3.7290

The least squares estimates are  $\hat{\beta}_0 = 8.46$  and  $\hat{\beta}_1 = -0.142$

- c. The estimated slope value  $\hat{\beta}_1 = -0.142$  indicates that for each 1 hour increase in the time between the capture of the fish and their placement in storage there is approximately a 0.142 decrease in the average quality of the fish.

11.10  $\hat{y} = 8.46 - 0.142x \Rightarrow \hat{y} = 8.46 - 0.142(10) = 7.04$

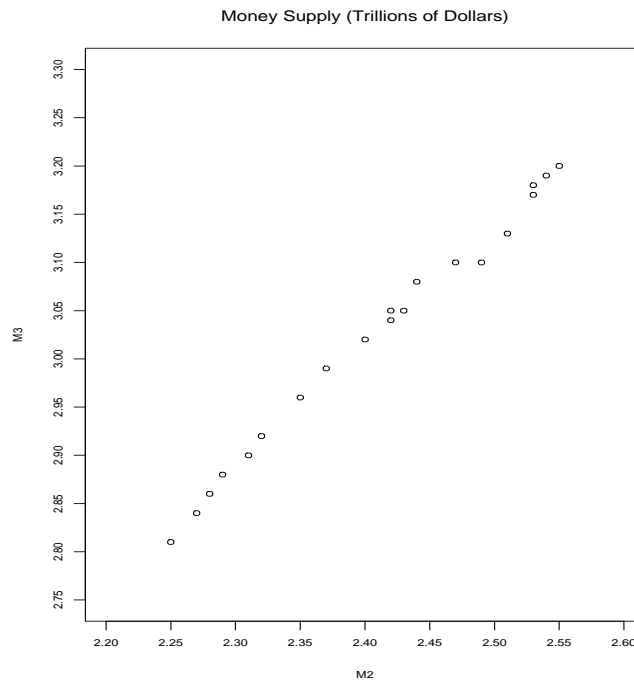
It would not be a good idea to make predictions at  $x = 18$

11.15 logarithmic transformation is suggested from the plot.

11.16 a. For the transformed data, the plotted points appear to be reasonably linear.

- b. The least squares line is  $\hat{y} = 3.10 + 2.76\sqrt{x}$   
 (Using rounded values  $3.097869 \approx 3.10$  and  $2.7633138 \approx 2.76$ ).

11.21 a. A scatterplot of the data is given here:



- b. The lifetime of one of the bits at a speed 100 is quite a bit smaller than the other three lifetimes at this speed.

11.22 a. The estimated intercept is 6.03 and the estimated slope is -0.017.

- b. The slope has a negative sign which indicates a decreasing relation between lifetime and speed.
- c. 0.6324.
- 11.23 a. The prediction equation is  $\hat{y} = 6.03 - 0.017x$ . Thus, we just replace  $x$  with the speeds of 60, 80, 100, 120, and 140 to obtain the values:

$$x = 60 \Rightarrow \hat{y} = 5.01$$

$$x = 80 \Rightarrow \hat{y} = 4.67$$

$$x = 100 \Rightarrow \hat{y} = 4.33$$

$$x = 120 \Rightarrow \hat{y} = 3.99$$

$$x = 140 \Rightarrow \hat{y} = 3.65$$

11.29 Minitab output is given here:

Regression Analysis: FIRMNESS versus CONCENTRATION

The regression equation is  
FIRMNESS = 48.9 + 10.3 CONCENTRATION

Predictor	Coef	SE Coef	T	P
Constant	48.933	1.541	31.76	0.000
CONCENTR	10.3333	0.7957	12.99	0.000

S = 2.387      R-Sq = 97.7%      R-Sq(adj) = 97.1%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	961.00	961.00	168.65	0.000
Residual Error	4	22.79	5.70		
Total	5	983.79			

a.  $\hat{y} = 48.9 + 10.3x$

b.  $S^2 = 5.70$

c.  $SE(\hat{\beta}_1) = 0.7957$

11.30  $H_o : \beta_1 = 0$  versus  $H_a : \beta_1 \neq 0$

Test Statistic:  $t = \frac{\hat{\beta}_1}{s_e/S_{xx}} = \frac{10.33}{2.387/9} = 12.99$

$p$ -value =  $2P(t_4 > 12.99) < 0.0001 \Rightarrow$  Reject  $H_o$

11.35 a. Yes

b.  $\hat{y} = 12.51 + 35.83x$

11.36 a. 1.069

b. 6.957

c.  $H_o : \beta_1 \leq 0$  versus  $H_a : \beta_1 > 0$

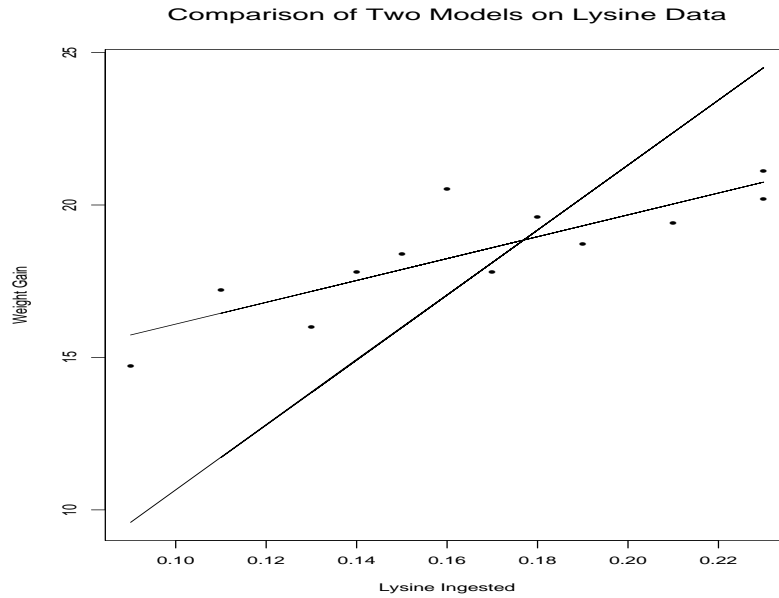
Test Statistic:  $t = 5.15$

$p$ -value =  $P(t_{10} > 5.15) = 0.0002 \Rightarrow$  Reject  $H_o$  and conclude there is significant evidence that there is a positive linear relationship.

11.37 a. No

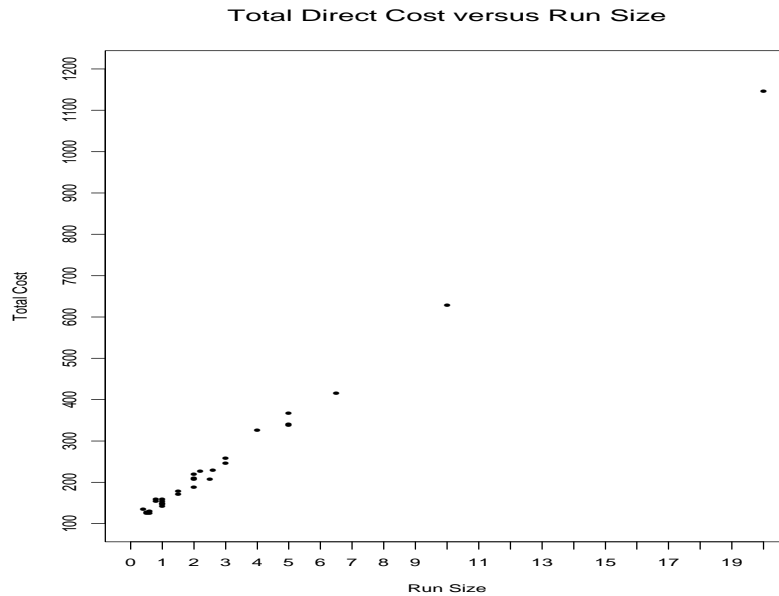
11.38 a.  $\hat{\beta}_1 = 106.52$

b. Scatterplot of the data and the two fitted lines are given here:



The model with an intercept term produced a better fit to the data values.

11.39 a. Scatterplot of the data is given here:



b.  $\hat{y} = 99.777 + 5.1918x$

The residual standard deviation is  $s = 12.2065$

c. A 95% C.I. for the slope is given by (5.072, 5.312)

11.43 95% prediction interval for log biological recovery percentage at  $x=30$  is given by (0.941, 1.449)

11.44 a.  $\hat{y} = -1.733333 + 1.316667x$

b. The p-value for testing  $H_o : \beta_1 \leq 0$  versus  $H_a : \beta_1 > 0$  is  
 $p\text{-value} = P(t_{10} \geq 6.342) < 0.0005 \Rightarrow \text{Reject } H_o$

11.45 a. 95% Confidence Intervals for  $E(y)$  at selected values for  $x$ :

$x = 4 \Rightarrow (2.6679, 4.3987)$

$x = 5 \Rightarrow (4.2835, 5.4165)$

$x = 6 \Rightarrow (5.6001, 6.7332)$

$x = 7 \Rightarrow (6.6179, 8.3487)$

b. 95% Prediction Intervals for  $y$  at selected values for  $x$  :

$x = 4 \Rightarrow (1.5437, 5.5229)$

$x = 5 \Rightarrow (2.9710, 6.7290)$

$x = 6 \Rightarrow (4.2877, 8.0456)$

$x = 7 \Rightarrow (5.4937, 9.4729)$

11.46 a.  $\hat{y} = 99.77704 + 51.9179x \Rightarrow$  When  $x=2.0$ ,  $\hat{E}(y) = 203.613$

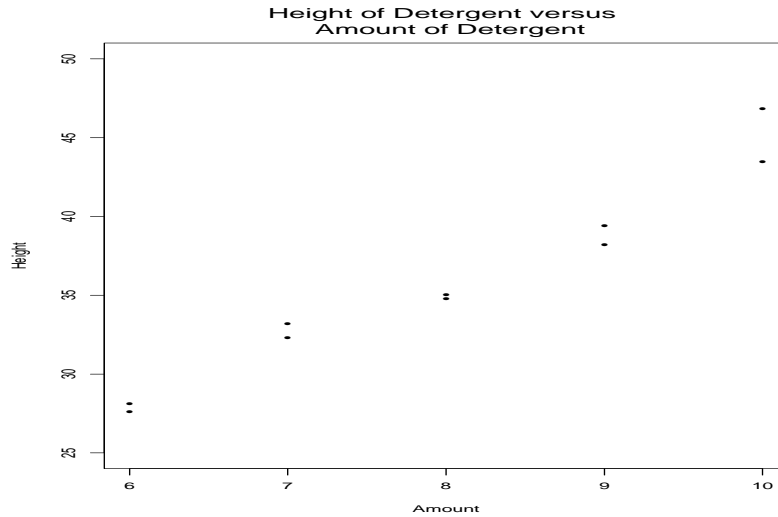
b. The 95% C.I. is given in the output as (198.902, 208.323)

11.47 No

11.48 a. The 95% P.I. is given in the output as (178.169, 229.057)

b. Yes

11.53 a. Scatterplot of the data is given here:



b. The SAS output is given here:

Model: MODEL1  
Dependent Variable: Y

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	330.48450	330.48450	169.213	0.0001
Error	8	15.62450	1.95306		
C Total	9	346.10900			

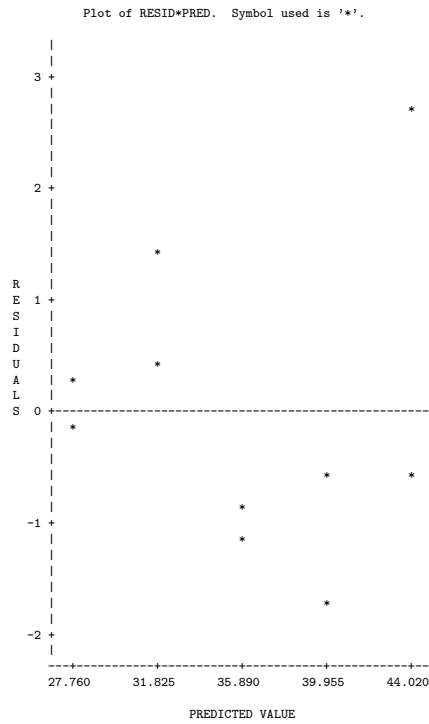
Root MSE	1.39752	R-square	0.9549
Dep Mean	35.89000	Adj R-sq	0.9492
C.V.	3.89390		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	3.370000	2.53872138	1.327	0.2210
X	1	4.065000	0.31249500	13.008	0.0001

$$\hat{y} = 3.37 + 4.065x$$

c. The residual plot is given here:



The residual plot indicates that higher order terms in  $x$  may be needed in the model.

11.54 a. A test of lack of fit is given here:

$$SSP_{exp} = 6.715$$

From the output of exercise 11.53,  $SS(\text{Residuals}) = 15.6245$ . Thus,

$$SS_{Lack} = 15.6245 - 6.715$$

$$df_{Lack} = 3$$

$$df_{exp} = 5$$

$$F = 2.21 < 5.41 = F_{.05,3,5}$$

There is not sufficient evidence that the linear model is inadequate.

b.  $S_{xx} = \sum_i (x_i - \bar{x})^2 = 20$ . Prediction interval for  $y$  at  $x = 6, 7, 8, 9, 10$

$x$	$\hat{y}$	95% P.I.
6	27.760	(24.086, 31.434)
7	31.825	(28.369, 35.281)
8	35.890	(32.510, 39.270)
9	39.955	(36.499, 43.411)
10	44.020	(40.346, 47.694)

11.57 a.  $\hat{y} = 0.113 + 0.118x$

Dependent variable is Transformed CUMVOL and Independent variable is Log(Dose).

b.  $\hat{x} = (y - 0.11277)/0.11847$   
 $\hat{x}_L = 2.40 + \frac{1}{(1-.2426)}(\hat{x} - 2.40 - d)$   
 $\hat{x}_U = 2.40 + \frac{1}{(1-.2426)}(\hat{x} - 2.40 + d) \Rightarrow$

y	TRANS(y)	$\widehat{LOG}(x)$	$\hat{x}$	d	$\widehat{LOG}(x)_L$	$\widehat{LOG}(x)_U$	$\hat{x}_L$	$\hat{x}_U$
10	.322	1.764	5.84	.242289	1.24038	1.88018	3.46	6.55
14	.383	2.285	9.83	.198634	1.98616	2.51068	7.29	12.31
19	.451	2.855	17.38	.221359	2.70876	3.29328	15.01	26.93

11.58 The four values of CUMVOL are 10, 20, 30, 12 yielding  $\bar{y} = 0.42969$  and  $s_{\bar{y}} = .05849$ , where y is the arcsine of the square root of CUMVOL.

For 50%:  $\hat{x} = 2.367$  and 95% confidence limits are (0.79, 4.45)

For 75%:  $\hat{x} = 5.861$  and 95% confidence limits are (2.20, 12.88)

11.59  $r = 0.9722$ .

11.60 a. Test  $H_o : \rho_{yx} \leq 0$  versus  $H_a : \rho_{yx} > 0$

Test Statistic:  $t_{13.13}$

$p - value < 0.0005$

Conclusion: There is significant evidence that the correlation is positive.

11.61 a.  $r_{yx}^2 = 99.64\%$  (R-squared on output).

b.  $r_{yx}$  should be positive.

11.63 a. There appears to be a general increase in salary as the level of experience increases.

11.65 a.  $\hat{y} = 40.507 + 1.470x$

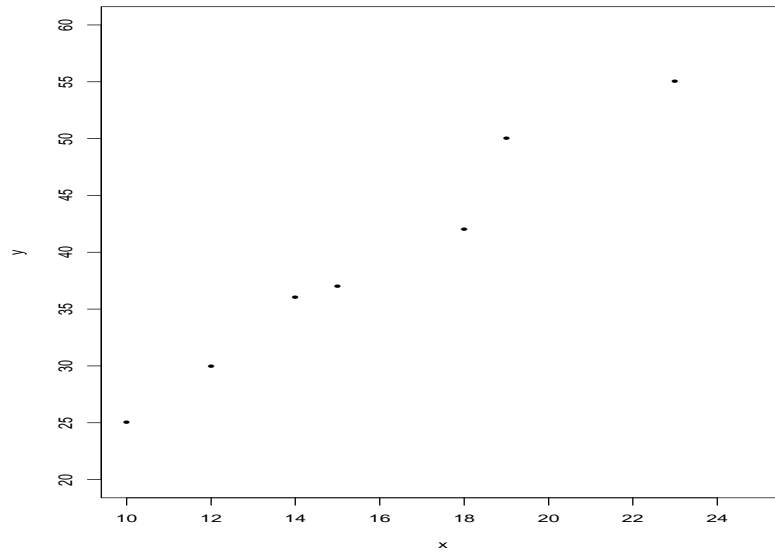
b. The residual standard deviation is  $s_{\epsilon} = 5.402$ .

c. From the output  $t = 6.916$  with a p-value of 0.000. This would imply overwhelming evidence that there is a relation between starting salary and experience.

d.  $R^2 = 0.494$

11.67 a. Scatterplot of the data is given here:

Plot of Data for Exercise 11.67



- b.  $\hat{\beta}_1 = 2.3498 \Rightarrow$   
 $\beta_0 = 2.0247 \Rightarrow \hat{y} = 2.2047 + 2.3498x$   
 c. When  $x_{n+1} = 21$ ,  $\hat{y} = 51.55$

11.68 a.  $s_\epsilon = 1.9583$   
 All the residuals fall within  $0 \pm 2s_\epsilon = (-3.9166, 3.9166)$

11.77 a.  $t = 2.14$ , with a p-value of 0.0396.

11.79 a.  $\hat{y} = 9.7709 + 0.0501x$  and  $s_\epsilon = 2.2021$   
 b. 90% C.I. on (0.0486, 0.0516)

11.81 There is an error in the Excel output given in the book. Output from Minitab is given here:

Regression Analysis: Rate: versus Mileage:

The regression equation is  
 Rate: = 9.79 + 0.0503 Mileage:

Predictor	Coef	SE Coef	T	P
Constant	9.7932	0.4391	22.30	0.000
Mileage:	0.0503229	0.0008230	61.14	0.000

S = 2.042      R-Sq = 98.8%      R-Sq(adj) = 98.8%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	15590	15590	3738.63	0.000
Residual Error	46	192	4		
Total	47	15782			

Predicted Values for New Observations

New Obs	Fit	SE Fit	95.0% CI	95.0% PI
1	26.903	0.298	( 26.303, 27.503)	( 22.749, 31.057)

Values of Predictors for New Observations

New Obs	Mileage:
1	340

$$\hat{y} = 26.903$$

The 95% P.I. when x=340 is (22.749, 31.057)

11.87 The Minitab output is given here:

Regression Analysis: Yield versus Nitrogen

The regression equation is  
 Yield = 11.8 + 6.50 Nitrogen

Predictor	Coef	SE Coef	T	P
Constant	11.778	1.887	6.24	0.000
Nitrogen	6.5000	0.8736	7.44	0.000

S = 2.140      R-Sq = 88.8%      R-Sq(adj) = 87.2%

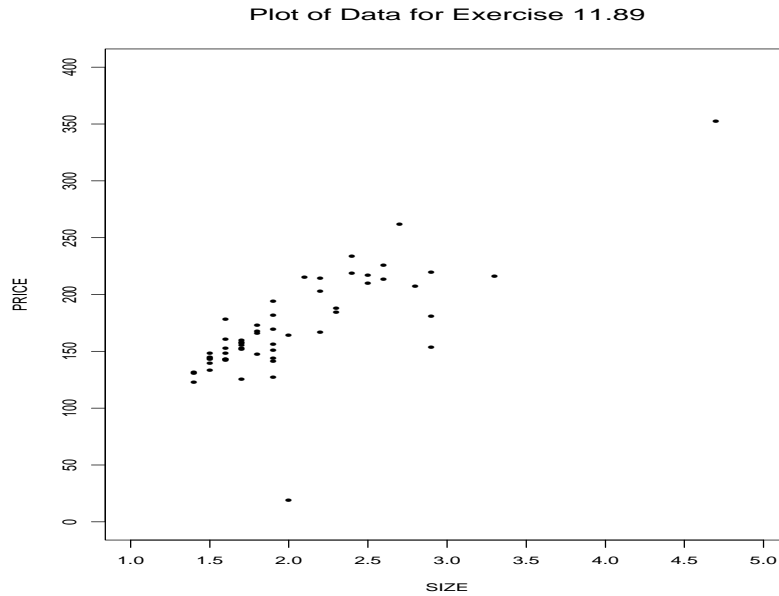
Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	253.50	253.50	55.36	0.000
Residual Error	7	32.06	4.58		
Lack of Fit	1	2.72	2.72	0.56	0.484
Pure Error	6	29.33	4.89		
Total	8	285.56			

$$\hat{y} = 11.778 + 6.5x$$

From the output,  $MS_{Lack} = 2.72$ ,  $MS_{exp} = 4.89 \Rightarrow F = \frac{2.72}{4.89} = 0.56$  with  $df = 1, 6$ . This yields  $p\text{-value} = 0.484$  which implies there is no indication of lack of fit in the model.

11.89 a. Scatterplot of the data is given here:



c. The minitab output is given here:

Regression Analysis: PRICE versus SIZE

The regression equation is  
 PRICE = 51.1 + 59.2 SIZE

Predictor	Coef	SE Coef	T	P
Constant	51.08	13.97	3.66	0.001
SIZE	59.152	6.666	8.87	0.000

S = 29.14                      R-Sq = 58.9%                      R-Sq(adj) = 58.1%  
 PRESS = 50352.0              R-Sq(pred) = 55.66%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	66862	66862	78.73	0.000
Residual Error	55	46707	849		
Total	56	113569			

Unusual Observations

Obs	SIZE	PRICE	Fit	SE Fit	Residual	St Resid
4	4.70	352.00	329.09	18.32	22.91	1.01 X
27	2.00	19.10	169.38	3.86	-150.28	-5.20R
47	2.90	154.00	222.62	7.06	-68.62	-2.43R

R denotes an observation with a large standardized residual  
 X denotes an observation whose X value gives it large influence.

Durbin-Watson statistic = 2.10

The least squares line is  $\hat{y} = 51.08 + 59.152x$

d. Minitab output for the data set without the outlier is given here:

Regression Analysis: PRICE versus SIZE

The regression equation is  
 PRICE = 54.0 + 59.0 SIZE

Predictor	Coef	SE Coef	T	P
Constant	53.99	10.06	5.37	0.000
SIZE	59.040	4.794	12.31	0.000

S = 20.96                      R-Sq = 73.7%                      R-Sq(adj) = 73.3%  
 PRESS = 26440.0              R-Sq(pred) = 70.73%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	66607	66607	151.64	0.000
Residual Error	54	23719	439		
Total	55	90327			

Unusual Observations

Obs	SIZE	PRICE	Fit	SE Fit	Residual	St Resid
4	4.70	352.00	331.48	13.18	20.52	1.26 X

13	2.90	181.00	225.20	5.09	-44.20	-2.17R
26	2.70	262.00	213.40	4.32	48.60	2.37R
46	2.90	154.00	225.20	5.09	-71.20	-3.50R

R denotes an observation with a large standardized residual  
 X denotes an observation whose X value gives it large influence.

Durbin-Watson statistic = 1.83

Predicted Values for New Observations

New Obs	Fit	SE Fit	95.0% CI	95.0% PI
1	349.19	14.59	( 319.94, 378.43)	( 297.99, 400.38) XX

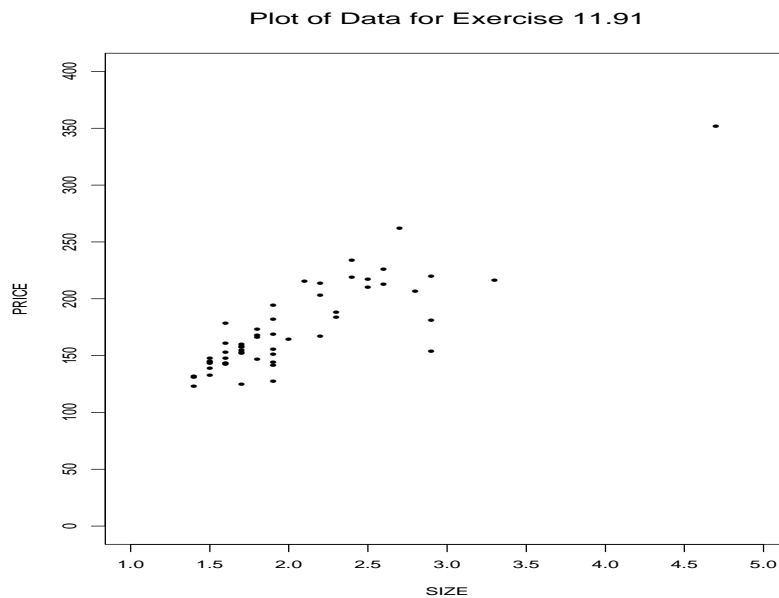
X denotes a row with X values away from the center  
 XX denotes a row with very extreme X values

Values of Predictors for New Observations

New Obs	SIZE
1	5.00

$$\hat{y} = 53.99 + 59.04x.$$

- e. The residual standard deviations are  $s_e = 29.14$  with outlier and  $s_e = 20.96$  without outlier.
- 11.90 a. The estimated intercept is  $\hat{\beta}_0 = 53.99$ .  
 c. Using the estimates from the Minitab output, a 95% C.I. for  $\beta_1$  is (49.428, 68.652).
- 11.91 a. From the Minitab output, a 95% P.I. for the selling price when the size is 5: (297.99, 400.38).  
 b. Scatterplot of the data is given here:



- c. The P.I. may not be valid since the statistical procedures depend on the condition that the variance remain constant across the range of x-values.

11.96 Minitab output is given here:

Regression Analysis: DURABIL versus CONCENTR

The regression equation is  
 DURABIL = 47.0 + 0.308 CONCENTR

Predictor	Coef	SE Coef	T	P
Constant	47.020	4.728	9.94	0.000
CONCENTR	0.3075	0.1114	2.76	0.008

S = 12.21                      R-Sq = 11.6%                      R-Sq(adj) = 10.1%  
 PRESS = 9368.62              R-Sq(pred) = 4.20%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	1134.7	1134.7	7.61	0.008
Residual Error	58	8644.5	149.0		
Total	59	9779.2			

Unusual Observations

Obs	CONCENTR	DURABIL	Fit	SE Fit	Residual	St Resid
2	20.0	25.20	53.17	2.73	-27.97	-2.35R
4	20.0	20.30	53.17	2.73	-32.87	-2.76R
60	60.0	18.90	65.47	2.73	-46.57	-3.91R

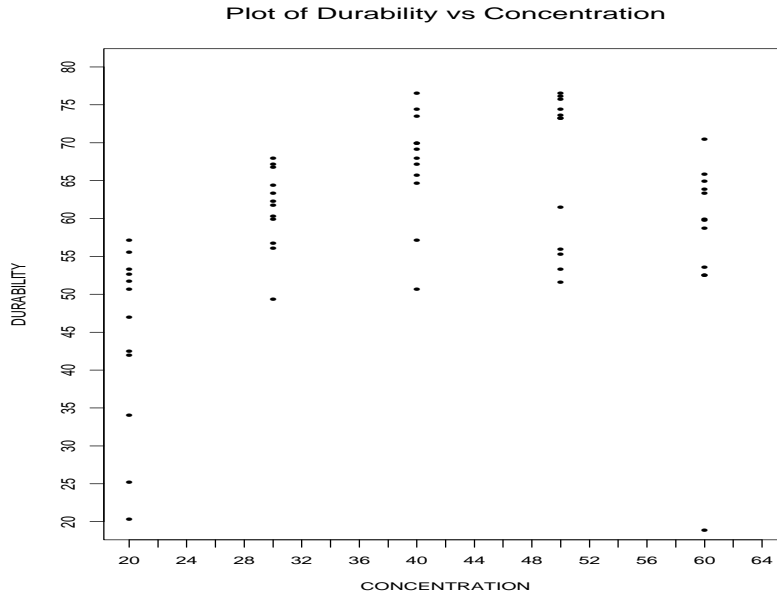
R denotes an observation with a large standardized residual

Durbin-Watson statistic = 1.35

- $\hat{y} = 47.020 + 0.3075x$ . The estimated slope  $\hat{\beta}_1 = 0.3075$  can be interpreted as follows: there is a 0.3075 increase in average durability for a 1 unit increase in the concentration.
- The coefficient of determination,  $R^2 = 11.6\%$ .

11.97 For testing  $H_o : \beta_1 = 0$  versus  $H_a : \beta_1 \neq 0$ ,  $t = 2.76$  with  $p$ -value = 0.008. The p-value is less than  $\alpha = 0.01$  thus we reject  $H_o$

11.98 Scatterplot of the data is given here:



- a. From the scatterplot, there is a definite curvature in the relation between Durability and Concentration. A straight-line model would not appear to be appropriate.
- b. The coefficient of determination,  $R^2$ , measures the strength of the linear (straight-line) relation only. A straight-line model does not adequately describe the relation between Durability and Concentration. This is indicated by the small percentage of the variation, 11.6%, in the values of Durability explained by the model containing just a linear relation with Concentration.

## Chapter 12: Multiple Regression and the General Linear Model

12.1 a.  $y_j = \mu_{i_j} + \epsilon_j$ , with  $i_j = 1, 2, \dots, 12$

$$b. \quad x_1 = \begin{cases} 1 & \text{if } V_1 = \text{1st Level} \\ 0 & \text{if } \text{Otherwise} \end{cases} \quad x_2 = \begin{cases} 1 & \text{if } V_2 = \text{1st Level} \\ 0 & \text{if } \text{Otherwise} \end{cases}$$

$$x_3 = \begin{cases} 1 & \text{if } V_2 = \text{2nd Level} \\ 0 & \text{if } \text{Otherwise} \end{cases} \quad x_4 = \begin{cases} 1 & \text{if } V_3 = \text{1st Level} \\ 0 & \text{if } \text{Otherwise} \end{cases}$$