Math 489 Test 1 L. Hogben Feb. 17, 2005

Read all 6 questions before you begin. Choose 5 (do not do all 6; I will grade only the first 5 I see). Each question is worth 15 points. All answers must be justified by computation or explanation. All electronic devices except calculators must be turned off. Calculators cannot contain any user-created program or other stored material.

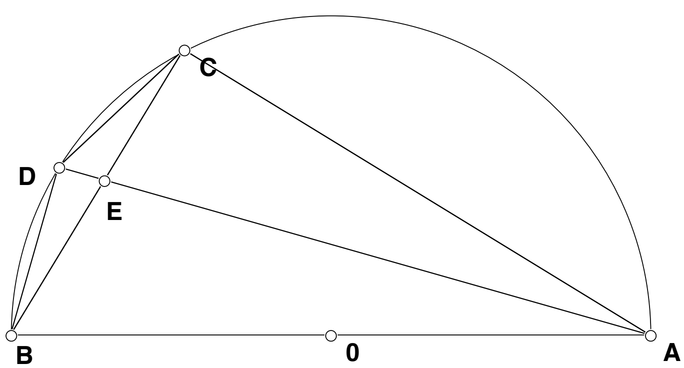
1. $N = 1 \pmod 2$, $N = 2 \pmod 3$, and $N = 1 \pmod 7$. Solve these congruences by a method used prior to 1100 CE to find a solution N (you need not find all solutions, nor do you need to find the smallest). State the culture in which your method was used.

2. Use al-Haytham's formula $(n+1)\sum_{i=1}^{n} i^k = \sum_{i=1}^{n} i^{k+1} + \sum_{p=1}^{n}\left(\sum_{i=1}^{p} i^k\right)$ and the formulas $\sum_{i=1}^{n} i^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$, $\sum_{i=1}^{n} i = \frac{n^2}{2} + \frac{n}{2}$ to find a formula for $\sum_{i=1}^{n} i^3$.

3. Use the figure below, which illustrates part of Archimedes argument evaluating π using an inscribed polygon. *ACB* is a semicircle with center *O*. *AD* is the bisector of angle *BAC* and *D* is on the semicircle. *E* is the intersection of *AD* and *BC*.

a) Give reasons why triangle *BAD* is similar to triangle *EAC* is similar to triangle *EBD*.

b) Give reasons why $\frac{AD}{DB} = \frac{AC}{CE} = \frac{BD}{DE}$



4. Summarize the evidence for transmission (if any) of Greek, Indian, and Chinese mathematics to Islamic mathematicians.

5. Briefly describe the differences between empirical mathematics, informal proof (also called demonstration), and formal proof (also called material axiomatics). For Greek culture and one other culture we have studied, compare and contrast which of these was considered mathematics and what was emphasized. Note any changes over time with each culture being discussed.

6. Summarize the development of the Hindu-Arabic number system, including the approximate time and the civilization in which important developments occurred.