

EXTRA CREDIT OPPORTUNITY:
Pythagorean Triples

A Pythagorean Triple (PT) is a triple (x, y, z) of positive integers such that $x^2 + y^2 = z^2$. We have shown that any PT has one of x, y even, the other odd, and has z odd. By convention, we assume x is even. The PT (x, y, z) is a *reduced triple* if x, y, z are pairwise relatively prime, i.e., $GCD(x, y) = GCD(y, z) = GCD(x, z) = 1$. We also assume (x, y, z) is a reduced triple, since any triple can be obtained from a reduced triple by multiplication. The goal is to describe the form of reduced triples.

Extra Credit Questions:

- a) Show that $(2pq, p^2 - q^2, p^2 + q^2)$ is a reduced triple if p, q are relatively prime positive integers of opposite parity.
- b) Show that for any triple (x, y, z) (reduced and x even), there exist p, q relatively prime positive integers of opposite parity such that $x = 2pq, y = p^2 - q^2, z = p^2 + q^2$.

Amount of Extra Credit:

- a) 1 point.
- b) Up to 10 points.

Rules:

- a) Must be done individually entirely by yourself- no outside sources, no discussion of problem with anyone else.
- b) May be done individually or with one partner. May not be discussed with any person who is not your partner. If outside source (e.g., book, web-site) is used, a xerox/printed copy of the relevant part of the source (with title/author or URL) must be provided. If a partner and/or outside source is used, the credit will be divided among the participants (including the source). Partial credit may be awarded for partial solution, but credit will awarded only for that which is correct.