

HW#3 (Math517)

1) 2.5.2 on Page 70.

Prove the following θ -scheme for heat equation is unconditionally stable with $\theta \geq \frac{1}{2}$.

$$(I - \theta k D_+ D_-) u_j^{n+1} = (I + (1 - \theta) k D_+ D_-) u_j^n$$

2) 2.5.3 on Page 70.

Derive the truncation error for the backward Euler and the Crank-Nicholson methods applied to $u_t = u_{xx}$. Prove that it is $O(h^2 + k)$ and $O(h^2 + k^2)$ respectively. Despite this fact, at certain times the backward Euler method is more accurate for the example computed in this section. Explain this paradox.

3) 2.7.1 on Page 76.

What explicit method could be used for the following Schrodinger type equation?

$$u_t = i u_{xx}$$

Derive the stability condition.