Quiz III (Math 165) **Sep 14th, 2007**

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1. Find out the discontinuous points for function $y=\dfrac{\sin(x)}{x(1-x)}$. Are they are removable? (1 point)

   Discontinuous points are $x=0$ and $x=1$, cause function does not have definition on them.

   $x=0$ is removable cause $\lim\limits_{x\to 0}\dfrac{\sin(x)}{x(1-x)}=1$ is a bounded value.

   $x=1$ is NOT removable cause $\lim\limits_{x\to 0}\dfrac{\sin(x)}{x(1-x)}=\infty$

2. Use **definition** to find out the derivative of function $y=\sqrt{x}$. (1.5 point)

$$y'=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})}$$

$$=\lim_{h\to 0}\frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{1}{(\sqrt{x+h}+\sqrt{x})}=\frac{1}{2\sqrt{x}}$$

3. Find out the derivatives. (6 points)

   1) $D_x(3x^3-4x^2-\dfrac{2}{x}+1)=9x^2-8x-2(-1)x^{-2}=9x^2-8x+\dfrac{2}{x^2}$

   2) $\dfrac{d}{dx}\left(\dfrac{5+2\alpha}{x}\right)=(5+2\alpha)(-1)x^{-2}=-\dfrac{5+2\alpha}{x^2}$

   3) $\left(x^3\cos(x)\right)'=(x^3)'\cos(x)+x^3(\cos(x))'=3x^2\cos(x)-x^3\sin(x)$

   4) $$\left(\frac{x^2-6x-5}{2x-3}\right)'=\frac{(x^2-6x-5)'(2x-3)-(x^2-6x-5)(2x-3)'}{(2x-3)^2}$$
   $$=\frac{(2x-6)(2x-3)-2(x^2-6x-5)}{(2x-3)^2}$$

   5) $$\left(\frac{3+\cos(x)}{x}\right)'=\frac{(3+\cos(x))'x-(3+\cos(x))(x)'}{x^2}=\frac{-\sin(x)x-(3+\cos(x))}{x^2}$$

   6) $\left(\sec(x)\right)'=\sec(x)\tan(x)$

4. Write out the equation of the tangent line for function $y=x^2-2x+1$ at point $(3,4)$. (1.5 point)

   Tangent line equation is written in format $y=m(x-x_0)+y_0$

   Given point is (3, 4), so we have $x_0=3, y_0=4$.

   The slope m is determined by the geometrical explanation of function derivative.

   $m=y'|_{x=3}=(2x-2)|_{x=3}=4$

   So we finally have $y=4(x-3)+4$