Math 166, Section A1 - Test 03

9 November 2007 Name: ____________________

DIRECTIONS: Answer the following questions or execute the following commands below. You may NOT use a calculator. Remember, you are an attorney and I am a jury of 12 people. You must convince me beyond a reasonable doubt that your answers are correct by showing work and *writing neatly*. **Should you have any questions**, do not hesitate to ask them.

1. Let $a_n = \dfrac{n}{n+1}$.

   (a) Does the sequence $\{a_n\}_{n=1}^{\infty}$ converge or diverge? If so, what does it converge to?

   (b) Does the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converge or diverge?

2. Use the ratio test to show that the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$ converges.

3. How did we define the term “convergence” for a series? (That is, what is the definition of the phrase “a series converges”?)

4. Use the ordinary comparison test to show that the series $\sum_{n=1}^{\infty} \frac{n}{2n^2 - 1}$ diverges.

5. Does the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$ converge conditionally, converge absolutely, or diverge? Justify your response with an argument that is clear, concise, and easy to follow.

6. List the three types of convergence sets for a power series with center a.

7. Chose **TWO AND ONLY TWO** of the following problems to complete. Space is provided for you on the following pages; clearly mark which two problems you are attempting.

   (a) Give an example of two divergent series $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ such that $\sum_{k=1}^{\infty}(a_k + b_k)$ converges.

   (b) How large must n be so that the error obtained by approximating the sum of the convergent series $\sum_{k=1}^{\infty} \frac{1}{k^4}$ with the n^{th} partial is no more than 0.0002?

   (c) Suppose that the power series $\sum_{n=0}^{\infty} a_n(x-3)^n$ converges at $x = -1$. Can we conclude that the series converges for $x = 6$? Can we conclude that the series converges for $x = 7$?

   (d) Recall the *Fibonacci sequence* $\{f_n\}$, given by

   $$f_1 = f_2 = 1, \qquad f_{n+2} = f_{n+1} + f_n.$$

   Determine the radius of convergence of the power series $\sum_{n=1}^{\infty} f_n(x-1)^n$.

   (e) Give a counterexample to show that the following statement is false: *If $a_n > 0$ for all natural numbers n and $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$.*

8. (Bonus points) Which two countries were the main combatants in the Spanish-American war? (You may write your answer below.)

Problem 7, part _________

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