Math 166, Section A1 - Test 02

14 September 2007 Name: _____

DIRECTIONS: Answer the following questions or execute the following commands below. You may NOT use a calculator. Remember, you are an attorney and I am a jury of 12 people. You must convince me beyond a reasonable doubt that your answers are correct by showing *all* of your work and *writing neatly*. **Should you have any questions**, do not hesitate to ask them.

(10 points) 1. Let $\alpha \geq 1$. Prove the reduction formula

$$\int (\ln x)^\alpha \, dx = x(\ln x)^\alpha - \alpha \int (\ln x)^{\alpha-1} \, dx.$$

(10 points) 2. Use the reduction formula in problem 1 to evaluate $\int (\ln x)^3 \, dx$.

(10 points) 3. A student is given the problem to find the length of the curve given by the parametric equations

$$x = t - \sin t,\ y = 1 - \cos t,\ 0 \le t \le 4\pi.$$

He correctly computes $x' = 1 - \cos t$ and $y' = \sin t$, then computes the integral

$$\int_0^{4\pi} \sqrt{(1-\cos t)^2 + \sin^2 t}\ dt.$$

He *correctly* computes the integral (the value is 0). Is he right or wrong? Justify your claim with an argument that is clear, concise, and easy to follow.

(30 points) 4. Evaluate the following antiderivatives.

(a) $\int \sin^7 x \, dx$

(b) $\int x(3x+10)^{49} \, dx$

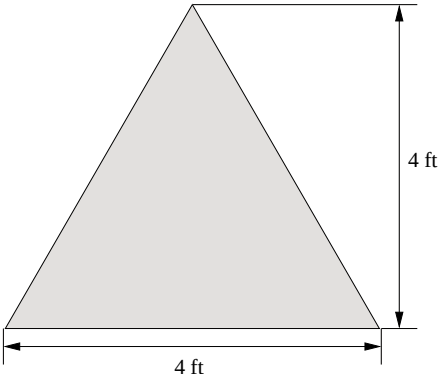
(c) $\int \arcsin x \, dx$

(d) $\int xe^x \, dx$

(e) $\int e^x \sin x \, dx$

(f) $\int x^2 \sin x \, dx$

(10 points) 5. A vertical cross section of a tank is shown in the figure below. Assume the tank is 10 feet long and filled to a height of 3 feet with a liquid with weight density δ pounds per cubic foot. Set up (but **DO NOT** evaluate) an integral to measure the work done in pumping the liquid to a height of 2 feet above the top of the tank.



6. (**Bonus: 5 points**.) What do "e.g." and "i.e." literally stand for?