Math 165, Section B, Summer Session II - Test 04

11 July 2008 Name: ____________________

DIRECTIONS: Answer the following questions or execute the following commands below. You may NOT use a calculator. Remember, you are an attorney and I am a jury of 12 people. You must convince me beyond a reasonable doubt that your answers are correct by showing *all* of your work and *writing neatly.* **Should you have any questions**, do not hesitate to ask them.

(5 points) 1. List the elements of the set $\{2\pi, 1, \sqrt[4]{3}, .7, -\pi+1, -2\}$ in increasing order.

Solution. $-\pi + 1 \leq -2 \leq .7 \leq 1 \leq \sqrt[4]{3} \leq 2\pi$. □

(15 points) 2. Let $f(x) = 3x^3 - 18x$. Find the intervals where f is increasing and concave up, increasing and concave down, decreasing and concave up, and decreasing and concave down.

Solution. We begin by finding the zeros of f' and f''. Since $f'(x) = 9x^2 - 18$ and $f''(x) = 18x$, we see that f' is zero for $x = \pm\sqrt{2}$ and f'' is zero for $x = 0$.

Since f' and f'' are both continuous, the intermediate value theorem guarantees that the sign of f' will be constant on the intervals $(-\infty, -\sqrt{2})$, $(-\sqrt{2}, \sqrt{2})$ and $(\sqrt{2}, \infty)$, and that the sign of f'' will be constant on the intervals $(-\infty, 0)$ and $(0, \infty)$.

Since $f'(-50) > 0$, $f'(0) < 0$, and $f'(50) > 0$, f is increasing on $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ and decreasing on $(-\sqrt{2}, \sqrt{2})$. Since $f''(-1) < 0$ and $f''(1) > 0$, f is concave down on $(-\infty, 0)$ and concave up on $(0, \infty)$.

Hence, f is increasing and concave up on $(\sqrt{2}, \infty)$, increasing and concave down on $(-\infty, -\sqrt{2})$, decreasing and concave up on $(0, \sqrt{2})$, and decreasing and concave down on $(-\sqrt{2}, 0)$. □

(5 points) 3. Is the point $x = 0$ an inflection point for the function $f(x) = x^4$? Why or why not?

Solution. No. $f''(0) = 0$, but since $f''(x) = 12x^2$, $f'' \geq 0$ for all x, and hence f is always concave up. □

(5 points) 4. Does the function $g(x) = \dfrac{1}{x}$ achieve a minimum on the interval $[-1, 3]$? What about on the interval $[1, 3]$? Why or why not?

Solution. The function g does not attain a minimum on the interval $[-1, 3]$ since for any $M < 0$, $g\left(\dfrac{1}{M-1}\right) = M - 1 < M$.

However, on $[1, 3]$, g is continuous and the interval $[1, 3]$ is closed; hence g must attain a minimum there. □

(10 points) 5. Show that for a rectangle of given perimeter K, the one with maximum area is a square.

Solution. Suppose we have a rectangle with sides x and y. Then $K = 2x + 2y$. We wish to find the maximum area of a rectangle under these conditions, and the area of such a rectangle is xy.

Since $y = \dfrac{K - 2x}{2}$, we need to maximize the function $f(x) = x \cdot \dfrac{K-2x}{2}$. $f'(x) = \dfrac{1}{2}(K - 4x)$, so the only critical point for f is $K/4$. Hence, the area is maximized when $x = K/4$. Since $y = \dfrac{K-2x}{2}$, substitution of $K/4$ for x shows that $y = K/4$. Hence, the area is maximized when $x = y = K/4$; i.e., when the rectangle is a square. □

(10 points) 6. Let $f(\theta) = \sin\theta$. Does the mean value theorem apply to f on the interval $[-\pi, \pi]$? Why or why not? If so, find all values of c such that $f'(c) = \dfrac{f(\pi) - f(-\pi)}{2\pi}$.

Solution. f is the sine function, and hence differentiable and continuous everywhere, so the mean value theorem does apply to f.

Since $f(-\pi) = f(\pi) = 0$, we need to find those points in the interval $[-\pi, \pi]$ for which $f'(x) = \cos(x) = 0$. These points are $\pi/2$ and $-\pi/2$. □