Math 165, Section B, Summer Session II - Test 03- Solutions

3 July 2008 Name: ____________________

DIRECTIONS: Answer the following questions or execute the following commands below. You may NOT use a calculator. Remember, you are an attorney and I am a jury of 12 people. You must convince me beyond a reasonable doubt that your answers are correct by showing *all* of your work and *writing neatly.* **Should you have any questions**, do not hesitate to ask them.

(10 points) 1. Assuming that each equation below defines a differentiable function of x, find y'.

(a) $x\sqrt{y+1} = xy + 1$

Solution. We take the derivative with respect to x of both sides of the equation to achieve:

$$\begin{aligned} \sqrt{y+1} + x \cdot \frac{1}{2}(y+1)^{-1/2} \cdot y' &= y + xy' \\ x \cdot \frac{1}{2}(y+1)^{-1/2} \cdot y' - xy' &= -\sqrt{y+1} + y \\ y' &= \frac{-\sqrt{y+1} + y}{x \cdot \frac{1}{2}(y+1)^{-1/2} - x} \end{aligned}$$

□

(b) $\cos(xy^2) = y^2 + x$

Solution. We take the derivative with respect to x of both sides of the equation to achieve:

$$\begin{aligned} -\sin(xy^2)(y^2 + 2yy'x) &= 2yy' + 1 \\ -2xy\sin(xy^2)y' - 2yy' &= y^2\sin(xy^2) + 1 \\ y' &= \frac{y^2\sin(xy^2) + 1}{-2xy\sin(xy^2) - 2y} \end{aligned}$$

□

(5 points) 2. If $f(x) = \sin(x^3)$, find $\dfrac{d^3f}{dx^3}$.

Solution. We first find $\dfrac{df}{dx}$, use that to find $\dfrac{d^2f}{dx^2}$, and finally use that to find $\dfrac{d^3f}{dx^3}$.

$$\frac{df}{dx} = \cos(x^3) \cdot 3x^2 = 3x^2\cos(x^3)$$

$$\frac{d^2f}{dx^2} = 6x\cos(x^3) + -\sin(x^3) \cdot 3x^2 \cdot 3x^2 = 6x\cos(x^3) - 9x^4\sin(x^3)$$

$$\begin{aligned} \frac{d^3f}{dx^3} &= 6\cos(x^3) + -\sin(x^3) \cdot 3x^2 \cdot 6x - 36x^3\sin(x^3) - \cos(x^3) \cdot 3x^2 \cdot 9x^4 \\ &= 6\cos(x^3) - 18x^3\sin(x^3) - 36x^3\sin(x^3) - 27x^6\cos(x^3) \\ &= 6\cos(x^3) - 54x^3\sin(x^3) - 27x^6\cos(x^3) \end{aligned}$$

□

(35 points) 3. Find the derivatives of each of the following functions.

(a) $f(x) = \sin(\cos(\tan(\sec(x^2))))$

Solution. Using the chain rule,

$$\frac{df}{dx} = \cos(\cos(\tan(\sec(x^2)))) \cdot [-\sin(\tan(\sec(x^2)))] \cdot \sec^2(\sec(x^2)) \cdot \sec(x^2)\tan(x^2) \cdot 2x$$

□

(b) $g(x) = \cos^3\left(\frac{x^2}{1-x}\right)$

Solution. Using the chain rule,

$$\frac{dg}{dx} = 3\cos^2\left(\frac{x^2}{1-x}\right) \cdot \left[-\sin\left(\frac{x^2}{1-x}\right)\right] \cdot \frac{(1-x)(2x) - x^2(-1)}{(1-x)^2}$$

□

(c) $h(x) = \cos^2(\cos(\cos t))$

Solution. (There is an obvious typo; t should be x.) Using the chain rule,

$$\frac{dh}{dx} = 2\cos(\cos(\cos x)) \cdot [-\sin(\cos(\cos x))] \cdot [-\sin(\cos x)] \cdot (-\sin x)$$

□

(d) $p(x) = x(1-x^2)^{13}$

Solution.

$$\frac{dp}{dx} = (1-x^2)^{13} + 13(1-x^2)^{12}(-2x) \cdot x$$

□

(e) $q(x) = \sqrt[4]{\sin x + x^8}$

Solution. We begin by rewriting $\sqrt[4]{\sin x + x^8}$ as $(\sin x + x^8)^{1/4}$. We have

$$\frac{dq}{dx} = \frac{1}{4} \cdot (\sin x + x^8)^{-3/4} \cdot (\cos x + 8x^7).$$

□

(f) $r(x) = \csc x$

Solution. Just memorization. $\frac{dr}{dx} = -\csc x \cot x$. □

(g) $s(x) = x\sin x + \frac{x^2-1}{3x^3 + \pi x^\pi}$

Solution. Using the product and chain rules,

$$\frac{ds}{dx} = \sin x + x\cos x + \frac{(3x^3 + \pi x^\pi)(2x) - (x^2-1)(9x^2 + \pi^2 x^{\pi-1})}{(3x^3 + \pi x^\pi)^2}$$

□

(Bonus points) 4. Which of A, B, C, D, and E are vowels?

A. B

B. E

C. D

D. A

E. C

F. B and D

G. A and E

Solution. This is obviously a trick question. There is no correct answer, depending on how one interprets the question. □