Name ____________________________

Note: Points for each question are indicated in the left margin in square brackets.

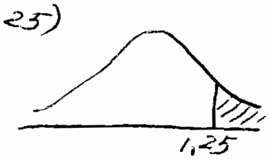
1. Suppose that volumes of liquid dispensed into bottles of certain type have mean 15.05 in^3 and standard deviation 0.04 in^3.

   (a) If volumes dispensed into bottles are approximately normally distributed, find the probability that the volume dispensed into the next bottle exceeds 15.1 in^3.

   [5] X: approximately normal with $\mu = 15.05$ and $\sigma = 0.04$.

   $P(X > 15.1) = P\left(Z > \frac{15.1 - 15.05}{0.04}\right) = P(Z > 1.25)$

   $= 1 - P(Z \le 1.25) = 1 - 0.8944 = 0.1056.$

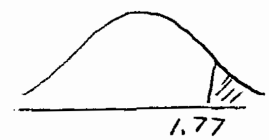


   (b) Approximate the probability that the average volume dispensed into the next 50 bottles exceeds 15.06 in^3.

   [5] $\bar{X}$: approximately normal with $\mu = 15.05$ and standard deviation $\sigma/\sqrt{n} = 0.04/\sqrt{50}$.

   $P(\bar{X} > 15.06) = P\left(Z > \frac{15.06 - 15.05}{0.04/\sqrt{50}}\right) = P(Z > 1.77)$

   $= 1 - 0.9616 = 0.0384$

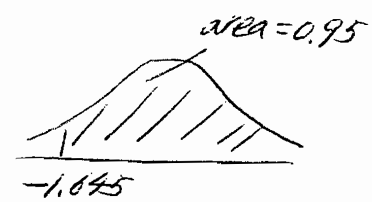


   (c) If it is desirable that the volume of liquid dispensed into a bottle exceeds 15 in^3 for 95% of bottles filled with liquid, what filling precision (as measured by standard deviation) is required if the mean volume of liquid dispensed into a bottle is kept at 15.05 in^3?

   [5] X: approximately normal, with mean $\mu = 15.05$.

   $P(X > 15) = P\left(Z > \frac{15 - 15.05}{\sigma}\right) = 0.95$

   $\Rightarrow \frac{15 - 15.05}{\sigma} = -1.645 \Rightarrow \sigma = 0.0304$



2. Suppose that X and Y are independent random variables with the same mean of 10 and standard deviation of 2. Find the mean and standard deviation of $W = X - Y + 2$.

[5] $EW = EX - EY + 2 = 10 - 10 + 2 = 2.$

$Var\ W = Var(X - Y + 2) = 1^2 \cdot Var\ X + (-1)^2 \cdot Var\ Y = 4 + 4 = 8.$

$\Rightarrow \sqrt{Var W} = \sqrt{8} = 2.828.$

3. Suppose that a large lot of nominal 10 Ω resistors has mean resistance $\mu = 9.95\ \Omega$ and standard deviation of resistances $\sigma = 0.06\ \Omega$. Suppose that 25 resistors are randomly selected from this lot and connected in series. Note that the resistance of the assembly is the sum of the resistances of the 25 individual resistors.

(a) Find the mean and variance for the resistance of the assembly.

[5] $X = X_1 + X_2 + \cdots + X_{25}$

$EX = EX_1 + EX_2 + \cdots + EX_{25} = 25\mu = 25 \cdot (9.95) = 248.75$

$\text{Var}X = \text{Var}X_1 + \text{Var}X_2 + \cdots + \text{Var}X_{25} = 25 \cdot \sigma^2 = 25(0.06)^2 = 0.09$

(b) Suppose that the resistance of the assembly, X, is normally distributed. Find $P(X < 250)$.

[5] X: normal with mean 248.75 and variance 0.09

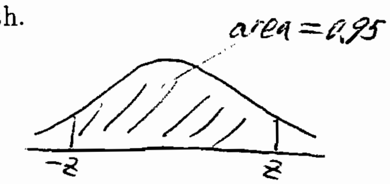
$P(X<250) = P\left(Z < \frac{250-248.75}{\sqrt{0.09}}\right) = P(Z < 4.17) \approx 1$

4. A sample of 90 U-bolts produced by a small company has thread lengths with a sample mean of 5.02 in. and a sample standard deviation of 0.003 in.

(a) Find a 95% two-sided confidence interval for the mean thread length.

[5] Use $\bar{X} \pm z \cdot \frac{s}{\sqrt{n}}$

$5.02 \pm 1.96 \cdot \frac{0.003}{\sqrt{90}}$, i.e., 5.02 ± 0.00062



(b) Find a 95% lower confidence bound for the mean thread length.

[5] Use $\bar{x} - z \cdot \frac{s}{\sqrt{n}}$

$5.02 - 1.645 \cdot \frac{0.003}{\sqrt{90}} = 5.0195.$

(c) Assess the strength of the evidence in these data to the effect that the population mean thread length differs from the nominal value of 5 in. Show the entire five-step format.

[10] ① $H_0: \mu = 5$

② $H_a: \mu \neq 5$

③ Test statistic: $z = \frac{\bar{x} - 5}{s/\sqrt{n}}$

Reference distribution: standard normal

Large values of $|z|$ are evidence for H_a.

④ The sample gives $z = \frac{5.02 - 5}{0.003/\sqrt{90}} = 63.25$.

⑤ p-value $= P(|Z| > 63.25) = 2 \cdot P(Z > 63.25) \approx 0$.

There is very strong evidence that the mean thread length differs from the nominal value of 5 in.